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Transformation of linear non-homogeneous differential equations of the second order to homogeneous

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ABSTRACT

The main results of this paper give a negative answer to the problem of transformations of the linear non-homogeneous differential equations of order two into a homogeneous one, by means of the internal elements of the non-homogeneous equation.

Other results, concerning perturbation by means of a sum with a certain mapping z and the exploitation of the methods are presented.

1. Introduction and preliminaries

The linear non-homogeneous differential equation of the second order is basic engineering equation for solving problems with internal resistances and external forces. It is believed that the equation is well elaborated. Fundamental tools for the equation are knowledge of homogeneous differential equation of the second order and Lagrange's method for variation of constants.

However, there is the need to control oscillations.The problem is how to perturb external force z in order to obtain desired behavior (a new oscillatoriness, monotonicity, limitedness of coefficients or solutions by various boundaries . . .).

The theorem on impossibility of transformation of non-homogeneous linear differential equation of the second order into a new homogeneous, by means of elements of the solution of the non-homogeneous equation $\{y_1 + Y_P, y_2 + Y_P\}$ where Y_P is a particular integral of the non-homogeneous equation is presented first. An important and necessary theorem on one and only Y_P for every solution of the homogeneous equation $C_1y_1 + C_2y_2$ where C_1 and C_2 are arbitrary constants is presented in what follows.

In previous theory and practice of differential equations, an equation is formed after the conditions of actual engineering problem, based on fundamental laws of physics. Very often the intention is to solve systems of differential equations by the method of perturbation of coefficients, which consists of replacement of parts of coefficients by small parameter ε, and solution is then treated by asymptotic methods with $\varepsilon \to 0$.

We are not aware that opposite has been done so far: for some subclasses of equations (for example non-homogeneous equation) to derive perturbations not over coefficients **but over solutions**, and to form new equations with desired

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